C2 Jan 2009 (MA)

Q1) $(3-2x)^5 \simeq (3)^5 + \binom{5}{1}(3)^4(-2x)^1 + \binom{5}{2}(3)^3(-2x)^2 + \ldots$

$\simeq 243 - 810x + 1080x^2$

Q2) $R = \int_{-1}^{4} (1+x)(4-x)\,dx = \int_{-1}^{4} [4 + 3x - x^2]\,dx$

$= \left[4x + \frac{3}{2}x^2 - \frac{x^3}{3}\right]_{-1}^{4} = \left[\frac{56}{3}\right] - \left[-\frac{13}{6}\right] = \boxed{\frac{125}{6}}$

Q3a)

x	1.8	2.2	2.6	3
y	3.84	4.14	4.39	4.58

b) $h = \frac{b-a}{n} = \frac{3-1}{5} = \frac{2}{5}$ //

Area $\simeq \frac{1}{2} \times \frac{2}{5}\left[3 + 4.58 + 2(3.47 + 3.84 + 4.14 + 4.39)\right]$

$\simeq \boxed{7.85}$

Q4) $\log_5(4-x) - \log_5(x^2) = 1$

$\log_5\left(\frac{4-x}{x^2}\right) = 1$

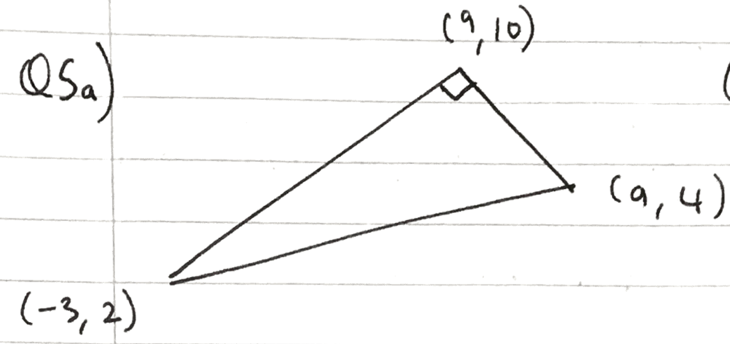
$5^1 = \frac{4-x}{x^2}$

$5x^2 = 4 - x$

$5x^2 + x - 4 = 0$ //

$(5x - 4)(x + 1) = 0 \quad \therefore \boxed{x = \frac{4}{5}}$ (reject $x = -1$ as $0 < x < 4$)

Q5a)



circle theorem –
(Angle in a semicircle is 90°)

$PQ^2 + QR^2 = PR^2$

$(-3-9)^2 + (2-10)^2 + (9-a)^2 + (10-4)^2 = (a+3)^2 + (4-2)^2$

$12^2 + 8^2 + (9-a)^2 + 36 = (a+3)^2 + 4$

$244 + 81 - 18a + \cancel{a^2} = 4 + \cancel{a^2} + 6a + 9$

$312 - 18a = 6a$

$24a = 312$

$\therefore a = \dfrac{312}{24} = \boxed{13}$

b) midpoint of PR : $\left(\dfrac{-3+13}{2}, \dfrac{2+4}{2}\right) = (5, 3)$ = Centre.

$|PR| = \sqrt{(13+3)^2 + (4-2)^2} = 2\sqrt{65}$ $\quad\therefore r = \sqrt{65}$

$\therefore r^2 = 65$

So $(x-5)^2 + (y-3)^2 = 65$

Q6a) $f(2) = r$ and $f(-1) = r$

$\therefore f(2) = f(-1)$.

$f(2) = (2)^4 + 5(2)^3 + 2a + b = 56 + 2a + b$

$f(-1) = (-1)^4 + 5(-1)^3 - a + b = -4 - a + b$

$\Rightarrow 56 + 2a + \cancel{b} = -4 - a + \cancel{b}$

$3a = -4 - 56 = -60$

$\therefore a = -20$

b) $\underline{f(-3) = 0}$: $(-3)^4 + 5(-3)^3 - 3a + b = 0$

$81 - 135 - 3a + b = 0$

$-54 - 3a + b = 0$

$a = -20$: $b = 54 + 3(-20)$

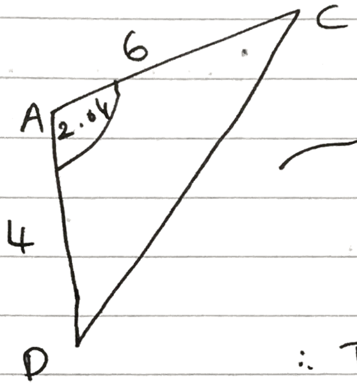
$\boxed{b = -6}$

Q7a) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2 \times 2.2 = \boxed{39.6\text{cm}^2}$

b) $2\pi - 2.2 = 4.083^c$ $\therefore$ $\angle DAC = \frac{4.083}{2} = \boxed{2.04^c}$

(Angles in a circle add up to 2π).
(around a point)

c)



Area ΔDAC = Area ΔDAB

$= \frac{1}{2}(6)(4)\sin(2.04) = \underline{\underline{10.69\text{cm}^2}}$

$\therefore$ Total area $= 10.69 + 10.69 + 39.6$
$= \boxed{61.0}$

Q8a) $4(1-\cos^2 x) + 9\cos x - 6 = 0$

$4 - 4\cos^2 x + 9\cos x - 6 = 0$

$4\cos^2 x - 9\cos x + 2 = 0$

b) $(4\cos x - 1)(\cos x - 2) = 0$

$\Rightarrow 4\cos x - 1 = 0$ | $\Rightarrow \cos x - 2 = 0$

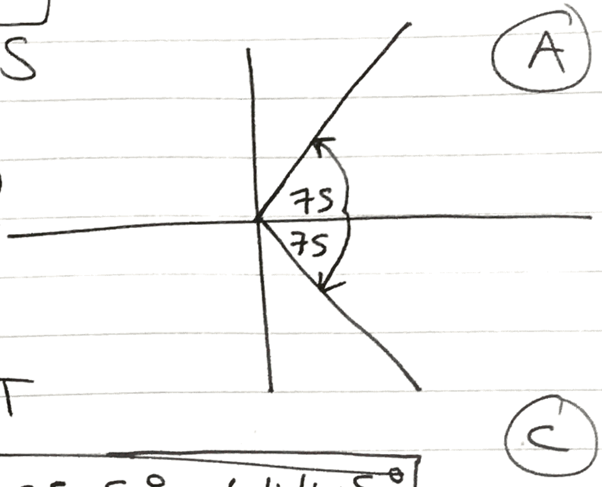
$\Rightarrow \cos x = \frac{1}{4}$ | $\Rightarrow \cos x = 2$ × reject. no valid solutions ($\cos x \leq 1$).

$x = \cos^{-1}\left(\frac{1}{4}\right) = 75.52°$ //

$\boxed{0 \leq x < 720^\circ}$

b(cont.)

$x = 75.5^\circ, (360 - 75.5^\circ),$
$(360 + 75.5^\circ),$
$(720 - 75.5^\circ)$



$\boxed{x = 75.5^\circ, 284.5^\circ, 435.5^\circ, 644.5^\circ}$

Q9a)

a	ar	ar^2
$(k+4)$	k	$2k-15$

$\frac{ar}{a} = r = \frac{k}{k+4}$ $\therefore ar^2 = (k+4) \times \frac{(k^2)}{(k+4)^2}$

so $\frac{\cancel{(k+4)}(k^2)}{(k+4)^{\cancel{2}}} = 2k - 15$

$\Rightarrow \frac{k^2}{k+4} = 2k - 15$

$\Rightarrow k^2 = (2k-15)(k+4)$

$\Rightarrow k^2 = 2k^2 + 8k - 15k - 60$

$\Rightarrow \underline{k^2 - 7k - 60 = 0}$

b) $(k-12)(k+5) = 0$

$k = 12$ or $k = -5$ ← reject as $k > 0$

$\therefore \boxed{k = 12}$

c) $r = \frac{k}{k+4} = \frac{12}{12+4} = \frac{12}{16} = \boxed{\frac{3}{4}}$

d) $S_\infty = \frac{a}{1-r} = \frac{k+4}{1-\frac{3}{4}} = \frac{12+4}{\frac{1}{4}} = \boxed{64}$

Q10a) Surface area $= A = 2\pi r^2 + 2\pi r h = 800$

$\therefore 2\pi r h = 800 - 2\pi r^2$

$\therefore h = \frac{800 - 2\pi r^2}{2\pi r}$

$V = \pi r^2 h = \cancel{\pi} r^{\cancel{2}} \left(\frac{800 - 2\pi r^2}{2 \cancel{\pi} \cancel{r}} \right) = \frac{800r - 2\pi r^3}{2}$

$V = \frac{800r}{2} - \frac{2\pi r^3}{2} = \boxed{400r - \pi r^3}$

b) $\frac{dV}{dr} = 400 - 3\pi r^2 = 0$

$\Rightarrow \frac{400}{3\pi} = r^2 \quad \therefore r = \sqrt{\frac{400}{3\pi}} = 6.515\ldots$

$\therefore V_{max} = 400(6.515) - \pi(6.515)^3 = \boxed{1737 \text{cm}^3}$

c) $\frac{d^2V}{dr^2} = -6\pi r < 0$ for all values of $r > 0$.

$\therefore$ Value of V previously found is a max.